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Book review

Mathematical Modeling in Continuum Mechanics, R. Temam and A. Miranville. Cambridge University Press, Cambridge, 2000

This relatively small book (288 pages gross) treats continuum mechanics as a whole. It begins with the kinematics and dynamics of rigid bodies, fluid mechanics, and after that considers solid mechanics. Its aim, as stated by the authors, is to introduce mathematical students into problems of mechanics.

The authors assumed that the goal could be achieved much better if the exposition were as similar to mathematical texts as possible. Accordingly, the text is replete with definitions, propositions, lemmas, theorems, etc. This style is definitely also familiar to a more mechanics – or physics-oriented readership, which, however, typically avoids it. It may nevertheless be helpful to mathematics students, who will be able to grasp the wide variety of topics covered by the book. These include elements of rheology, potential inviscid incompressible flows, compressible trans-sonic flows, linear acoustics, viscous flows, heat transfer, elements of the boundary layer theory and turbulence, magnetohydrodynamics, combustion theory, geophysical flows, linear elasticity with a number of classical examples on beam bending and torsion, and composite materials. The part on elasticity leads to such questions as deviations from linearity, nonlinear elasticity and plasticity (the latter, however, is not mentioned explicitly). The last part of the book deals with linear and nonlinear waves, including single vibrations, beats and packets of waves, the KdV-equation and its solitons, and the nonlinear Schrodinger equation.

The list of topics is very long, and apparently opens wide horizons for a student-reader. I found the expositions mostly clear. In spite of my own physical, rather than mathematical background, I was not seriously upset when some of the familiar facts became lemmas, theorems, etc. However, in a number of places I felt that emphasis is on those aspects which do not really concern a fluid mechanicist or an applied physicist (or even an applied mathematician).

In some places the mathematical accuracy of the derivation is not followed by a mechanical one. For example, in Section 3.4 the symmetry of the stress tensor is proved as though it were true for all materials. The fact that distributed moments of forces should be assumed to be zero, is not mentioned. Thus a reader could be surprised by liquid crystals, where the stress tensor is not symmetric (they are so widespread today, that inevitably some of the students have them in their watches).

In addition to the thermodynamical and phenomenological principles of rheology listed on p. 68, I would have preferred to see a mention of statistical physics as one of the main tools in search for rheological constitutive equations. Melted plastics are mentioned as one of the possible applications on p. 71, but the nonlinear viscoelasticity characteristic of them is actually not considered. When introducing some common laws, the authors omit the names associated with them, such as the *Newton–Stokes* or *Hooke* constitutive equations (pp. 70 and 72), or the *Fourier* law (p. 81) – contrary to common practice in the literature. In the same context it might be also instructive to demonstrate to mathematics students that some of the classics of mathematics were equally active in continuum mechanics, and succeeded in both fields.

Regarding linearity in problems, say, of acoustics (p. 120) it might be worth mentioning that linearity of the boundary conditions (not only of the equations) is needed.

It is a pity that the Prandtl equations and their self-similar solutions did not find their way into the discussion of the boundary layers. These topics involve a lot of in-depth mathematics which deserves mentioning. On the other hand, mere listing of the equations of the combustion theory as in Chapter 11, does not bring out its mathematical fascination. From my point of view, the particular fact from the theory of thermal explosion, namely that the simple ODE

$$\frac{d^2\theta}{d\xi^2} + 2e^{\theta} = 0$$

with the boundary conditions

$$\theta = 0 \quad \text{at } \xi = \xi_0$$

$$\frac{d\theta}{d\xi} = 0 \quad \text{at } \xi = 0$$

does not have a solution if ξ_0 is larger than 0.66 (which corresponds to a thermal explosion and can be proved analytically), sheds much more light on the combustion theory than the too dry explanation of Chapter 11. I also found it surprising that such a basic reference in the subject as Ya. B. Zeldovich et al.'s "The Mathematical Theory of Combustion and Explosions", New York, 1985 was not mentioned and taken into account in this context.

Incorporation of magnetohydrodynamics as one of the topics in an introductory course (Chapter 10) is an attractive effort. Some physical restrictions on the closure laws for the Maxwell equations (cf. p. 145) should be mentioned, at least so as to demonstrate the range of validity of the whole set. In a short introductory course like the present book, I found it surprising that such a particular topic as the Tokamak machine has been squeezed in. On the other hand, it is a pity that potential flows with free surfaces and creeping flows incorporating both interesting physical (e.g. surface tension) and mathematical (e.g. the integral Fredholm equations) topics were totally omitted.

In the treatment of beam bending on p. 202, I would have preferred to find a mention of the hypothesis of flat cross-sections, which is actually used implicitly.

Minor technicalities: on pp. 92 and 93 vectors are denoted as scalars, whereas on p. 96 they are denoted with arrows. It would be preferable to use uniform notation for vectors throughout the book, e.g. bold characters. In the first line on p. 91 ϕ should be replaced by 0; there is a graphical problem with the velocity profile in Fig. 9. 4b. On pp. 122 and 129 the term "thermohydraulics" is used, in my opinion, "heat transfer" would be preferable.

Summarizing, I find the book an interesting attempt at exposing mathematics students to problems of continuum mechanics. I hope that some of the students will read the book in depth. This, combined with the other sources, will allow them to confront the modern questions of continuum mechanics, which are (in many cases) fascinating mathematically and important for applications.

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